Control of Quadrotor Aerial Vehicles Equipped with a Robotic Arm

G. Arleo, F. Caccavale, G. Muscio and F. Pierri

Abstract—In this paper a novel hierarchical motion control scheme for quadrotor aerial vehicles equipped with a manipulator is proposed. The controller is organized into two layers: in the top layer, an inverse kinematics algorithm computes the motion references for the actuated variables; in the bottom layer, a motion control algorithm is in charge of tracking the motion references computed by the top layer. A simulation case study is developed to demonstrate the effectiveness of the approach in the presence of disturbances and unmodeled dynamics.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) are being widely used in a number of applications, mostly military but also civilian, involving surveillance of indoor or outdoor environments, remote inspection and monitoring of hostile environments. Among UAVs, quadrotor helicopters are emerging as a popular platform, due to their larger payload capability and their higher maneuverability with respect to single-rotor vehicles.

Motion control of quadrotors is a widely investigated but still challenging issue, since the quadrotor is an under-actuated system and, often, is equipped with limited sensing devices. Conventional approaches to UAVs control have been based on linear controller design. These approaches include proportional integral derivative (PID) controllers [1], [2], proportional derivative controllers (PD) [3], linear quadratic regulator (LQR) controllers [4], [5] and robust $H_{\infty}$ controllers, requiring model linearization about a set of equilibrium conditions [6]. Linear controllers are characterized by satisfactory performance near the design conditions and in hovering conditions, but performance degradation is usually observed when the aircraft moves away from these conditions. To overcome these drawbacks, many nonlinear controllers have been proposed, such as model predictive control [7], adaptive control [8], backstepping and sliding mode techniques [9], [10]. In [11] the problem of actuator saturation has been taken into account via a nested saturation-based controller. Since it is possible to make a conceptual separation between position and orientation of a quadrotor, hierarchical controllers, based on an inner-outer loop, have been successfully proposed, for example, in [12], [13]. Finally, a number of vision based controllers have been adopted, see e.g. the results in [14], [15].

Recently, UAVs have been proposed for tasks as grasping and manipulation, this is a challenging issue since the vehicle is characterized by an unstable dynamics and the presence of the object causes nontrivial coupling effects [16]. Moreover, the capability to perform complex tasks requires suitable mechanical structure: in [17], [18] properly designed grippers for aerial grasping processes are proposed, while [19] presents an UAV equipped with a robotic arm to execute dexterous manipulation tasks. In detail, in [19] the dynamic model of the whole system, UAV plus manipulator, is devised and a Cartesian impedance control is developed in such a way to cope with contact forces and external disturbances.

In this paper, the problem of motion control of the end-effector of a robot manipulator mounted on a quadrotor helicopter is tackled through a hierarchical control architecture. Namely, in the top layer, an inverse kinematics algorithm computes the motion references for the actuated variables, i.e., position and yaw angle of the quadrotor vehicle and joint variables for the manipulator. In the bottom layer, a motion control algorithm is in charge of tracking the motion references. The proposed motion controller in the bottom layer is an extension of the approach proposed in [12] to vehicle-manipulator systems: a vehicle position controller computes the thrust force and the reference values for pitch and roll angles, then an attitude controller, on the basis of these references, computes the moments acting on the quadrotor, while a manipulator controller computes the joint torques.

A simulation case study is developed to demonstrate the effectiveness of the approach in the presence of disturbances and unmodeled dynamics.

II. MODELING

A quadrotor is a rotorcraft with four rotors; the propeller of each rotor generates a force and the total thrust is given by the sum of the individual forces. Therefore, the quadrotor is an under-actuated vehicle with four input forces and six degrees of freedom (DOFs), describing the position and orientation of the vehicle center of mass. Two of the rotors rotate in clockwise direction, while the other two are counterclockwise.

The system considered in this paper, depicted in Fig. 1, is composed by a quadrotor vehicle equipped with a $n$-DOF robotic arm.

A. Kinematics

Let $\Sigma_b$ denote the vehicle body-fixed reference frame with origin at the vehicle center of mass; its position with respect to the world fixed inertial reference frame, $\Sigma$, is given by the $(3 \times 1)$ vector $p_b$, while its orientation is given by the rotation matrix $R_b$.

$$
R_b(\phi_b) = \begin{bmatrix}
    c\theta c\phi & c\theta s\phi s\theta - s\phi c\theta & c\phi s\theta c\phi + s\phi s\theta \\
    s\phi s\theta & s\phi c\theta c\phi + s\phi s\theta & -s\phi s\theta c\phi + c\phi s\theta \\
    -s\theta & c\theta c\phi & c\theta s\phi
\end{bmatrix},
$$

(1)

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where $\phi_b = [\psi \theta \phi]^T$ is the triple of ZYX yaw-pitch-roll angles and $c_\psi$ and $s_\psi$ denote, respectively, $\cos \gamma$ and $\sin \gamma$.

Let us consider the frame $\Sigma_e$ attached to the end effector of the manipulator. The position of $\Sigma_e$ with respect to $\Sigma$ is given by

$$p_e = p_b + R_b p_{eb}^b,$$  \hfill (2)

where the vector $p_{eb}^b$ describes the position of $\Sigma_e$ with respect to $\Sigma_b$. The linear velocity $\dot{p}_e$ of $\Sigma_e$ in the fixed frame is obtained by differentiating (2)

$$\dot{p}_e = \dot{p}_b - S(R_b p_{eb}^b) \omega_b + R_b \dot{p}_{eb}^b,$$  \hfill (3)

where $S(\cdot)$ is the $(3 \times 3)$ skew-symmetric matrix operator performing the cross product [20]. The orientation of $\Sigma_e$ can be described by the rotation matrix

$$R_e = R_b R_e^b,$$  \hfill (4)

where $R_e^b$ describes the orientation of $\Sigma_e$ with respect to $\Sigma_b$. By differentiating (4), the angular velocity $\omega_e$ of $\Sigma_e$ is given by

$$\omega_e = \omega_b + R_b \omega_{eb}^b,$$  \hfill (5)

where $\omega_{eb}^b = R_b^T (\omega_e - \omega_b)$ is the relative angular velocity between the end effector and the frame $\Sigma_b$, expressed in the frame $\Sigma_b$.

Let $q$ be the $(n \times 1)$ vector of joint coordinates of the manipulator. Then, $p_{eb}^b(q)$ and $R_e^b(q)$ represent the usual direct kinematics equations of a ground-fixed manipulator with respect to its base frame, $\Sigma_b$. The $(6 \times 1)$ vector of the generalized velocity of the end-effector with respect to $\Sigma_b$, $\dot{q} = \left[ \dot{p}_{eb}^b \omega_{eb}^b \right]^T$, can be expressed in terms of the joint velocities $\dot{q}$ via the manipulator Jacobian $J_{eb}^b$, i.e.,

$$\dot{q} = J_{eb}^b(q) \dot{q}. \hfill (6)$$

On the basis of (3) and (5), the generalized end-effector velocity, $v_e = \left[ \dot{p}_b^T \omega_b^T \right]^T$, can be expressed as

$$v_e = J_b(q, R_b) \dot{p}_b^b + J_{eb}(q, R_b) \dot{q}, \hfill (7)$$

where $v_b = \left[ \dot{p}_b^T \omega_b^T \right]^T$ and

$$J_b = \begin{bmatrix} I_3 & -S(R_b p_{eb}^b) \\ O_3 \end{bmatrix}, \quad J_{eb} = \begin{bmatrix} R_b & O_3 \\ O_3 & R_b \\ J_{eb}^b \end{bmatrix},$$

while $I_m$ and $O_m$ denote $(m \times m)$ identity and null matrices, respectively.

If the attitude of the vehicle is expressed in term of yaw-pitch-roll angles, equation (7) becomes

$$v_e = J_b(q, \phi_b) T_A(\phi_b) \dot{x}_b + J_{eb}(q, R_b) \dot{q}, \hfill (8)$$

with

$$x_b = \begin{bmatrix} p_b \\ \phi_b \end{bmatrix}, \quad T_A(\phi_b) = \begin{bmatrix} I_3 & O_3 \\ O_3 & T(\phi_b) \end{bmatrix},$$  \hfill (9)

where $T(\phi_b)$ is the transformation matrix between the angular velocity $\omega_b$ and the time derivative of the Euler angles $\phi_b$.

Since the vehicle is an under-actuated system, i.e., only 4 independent control inputs are available against the 6 degrees of freedom, the position and the yaw angle are usually the controlled variables, while pitch and roll angles are used as intermediate control inputs for position control. Hence, it is worth rewriting the vector $x_b$ as follows

$$x_b = \begin{bmatrix} \eta_b \\ \sigma_b \end{bmatrix}, \quad \eta_b = \begin{bmatrix} p_b \\ \psi \end{bmatrix}, \quad \sigma_b = \begin{bmatrix} \theta \\ \phi \end{bmatrix}.$$  \hfill (10)

Thus, the differential kinematics (8) becomes

$$v_e = J_{\eta}(q, \phi_b) \dot{\eta}_b + J_{\sigma}(q, \phi_b) \dot{\sigma}_b + J_{eb}(q, \phi_b) \dot{q}, \hfill (11)$$

where $\zeta = [\eta_b^T \sigma_b^T]^T$ is the vector of controlled variables, $J_{\eta}$ is composed by the first 4 columns of $J_b T_A(\phi_b)$, $J_{\sigma}$ is composed by the last 2 columns of $J_b T_A(\phi_b)$ and $J_{\zeta} = [J_{\eta} J_{eb}]$.

If the end-effector orientation is expressed via a triple of Euler angles (e.g., yaw-pitch-roll angles), $\phi_e$, the differential kinematics (10) has to be rewritten in terms of the vector $\dot{x}_e = \left[ \dot{p}_e^b \dot{\phi}_e \right]^T$. Indeed, since $\dot{x}_e = T_A^{-1}(\phi_e) v_e$, the differential kinematics becomes

$$\dot{x}_e = T_A^{-1}(\phi_e) \begin{bmatrix} \dot{J}_{\zeta}(\sigma_b, \zeta) \zeta + J_{\sigma}(\sigma_b, \zeta) \sigma_b \end{bmatrix}. \hfill (11)$$

**B. Dynamics**

The dynamical model of the vehicle-manipulator system can be derived, assuming negligible aerodynamical effects and low speed displacements, by using, e.g., the Euler-Lagrange formulation [19]

$$M(\xi) \ddot{\xi} + C(\xi, \dot{\xi}) \dot{\xi} + g(\xi) = u,$$  \hfill (12)

where $\xi = [x_b^T \dot{q}^T]^T \in \mathbb{R}^{(6+n \times 1)}$, $M$ represents the symmetric and positive definite inertia matrix of the system.
$C$ is the matrix of Coriolis and centrifugal terms, $g$ is the vector of gravity forces, $u$ is the vector of inputs

$$u = \begin{bmatrix} u_k \\ u_\mu \\ u_r \end{bmatrix} = \begin{bmatrix} R_b(\phi_b) f_b^T \\ R_b^T(\phi_b) T(\phi_b) \mu_b^T \end{bmatrix}[13]$$

$\tau$ is the $(n \times 1)$ vector of the manipulator joint torques, while the $(3 \times 1)$ vectors $f_b^T$ and $\mu_b^T$ are, respectively, the forces and the torques generated by the 4 motors of the quadrotor helicopter, expressed in the frame $\Sigma_b$.

The matrices introduced in (12) can be detailed by considering the expressions derived in [19]. In particular, the inertia matrix can be viewed as a block matrix

$$M(\xi) = \begin{bmatrix} M_{pp} & M_{pq} \\ M_{pq}^T & M_{q} \end{bmatrix},$$

where $M_{pp} \in \mathbb{R}^{3 \times 3}$, $M_{pq} \in \mathbb{R}^{3 \times n}$, $M_{\phi\phi} \in \mathbb{R}^{3 \times 3}$, $M_{\phi q} \in \mathbb{R}^{3 \times n}$ and $M_{qq} \in \mathbb{R}^{n \times n}$. The detailed expressions of the blocks are given in [19].

Similarly, matrix $C$ and vector $g$ in (12) can be detailed as

$$C(\xi, \dot{\xi}) = \begin{bmatrix} C_p \\ C_\phi \\ C_q \end{bmatrix}, \quad g(\xi) = \begin{bmatrix} g_p \\ g_\phi \\ g_q \end{bmatrix},$$

with $C_p \in \mathbb{R}^{3 \times (6+n)}$, $C_\phi \in \mathbb{R}^{3 \times (6+n)}$, $C_q \in \mathbb{R}^{n \times (6+n)}$ and $g_p \in \mathbb{R}^3$, $g_\phi \in \mathbb{R}^3$ and $g_q \in \mathbb{R}^n$.

The input vectors $f_b^T$ and $\mu_b^T$ can be expressed as

$$f_b^T = \begin{bmatrix} 0 \\ 0 \\ f_z \end{bmatrix}, \quad \mu_b^T = \begin{bmatrix} \mu_\psi \\ \mu_\phi \\ \mu_\mu \end{bmatrix},$$

where $f_z$ is the total thrust applied by the rotors along the $z_b$ axis. Both $f_z$ and $\mu_b^T$ are related to the four actuation forces output by the quadrotor motors $f$ via the following relation [21]

$$\begin{bmatrix} f_z \\ \mu_b^T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & l & 0 & -l \\ -l & 0 & l & 0 \\ c & -c & c & -c \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = N f,$$  (15)

where $l > 0$ is the distance from each motor to the vehicle center of mass, $c = \gamma_d/\gamma_\tau$, and $\gamma_d, \gamma_\tau$ are the drag and thrust coefficient, respectively.

### III. CONTROL SCHEME

A task for the vehicle-manipulator system modeled in the previous section is usually specified in terms of a desired trajectory for the end-effector position, $p_{e,d}(t)$, and orientation, $\phi_{e,d}(t)$; also, the corresponding desired linear and angular velocities, $\dot{p}_{e,d}(t)$ and $\dot{\phi}_{e,d}(t)$, are assigned. The proposed hierarchical control strategy is composed by two layers. In the top layer, an inverse kinematics algorithm computes the motion references $\eta_b(t), \eta_d(t), q_b(t)$ and $q_d(t)$. Then, in the bottom layer, a motion control algorithm is designed in such a way that $\eta_b$ and $q$ track the corresponding desired trajectories output by the top layer. The whole scheme is summarized in Fig. 2. It is assumed that $\dim\{\zeta\} = n + 4 \geq \dim\{\nu_e\} = 6$, i.e., the number of DOFs characterizing the vehicle-manipulator system is, at least, equal to the dimension of the assigned task.

### A. Inverse Kinematics

The differential kinematics (11) is considered to derive a closed-loop inverse kinematics algorithm [20], in charge of computing the motion references for the motion control loops placed at the bottom layer

$$\dot{\zeta}_r = J^T(\sigma_b, \zeta_r) T_A(\dot{\phi}_{e,r}) (\dot{x}_{e,d} + K e) - J^T(\sigma_b, \zeta_r) J_b(\sigma_b, \zeta_r, \dot{\sigma}_b),$$  (16)

where $J^T = J^T (J^T J^T)^{-1}$ is a pseudoinverse of $J$, $K$ is a symmetric positive definite gain matrix, $e$ is the kinematic inversion error, given by $e = x_{e,d} - x_{e,r}$, and $x_{e,r}$ is the end-effector pose computed on the basis of $\zeta_r$ and $\sigma_b$.

Notice that all the direct kinematics quantities are computed via (2) and (4) by using $\zeta_r$ (i.e., the output of the algorithm) and the measured value of $\sigma_b$, which is not computed by the algorithm.

If $n + 4 = 6$, the vehicle-manipulator system is non-redundant, namely $J^T$ is square and the pseudo-inverse can be replaced by the inverse. On the other hand, if $n + 4 > 6$ the vehicle-manipulator system is kinematically redundant and the redundant DOFs can be exploited to fulfill secondary tasks by resorting to a task-priority approach [22], [23].

**Remark 1:** If the motion control algorithms need the desired values of the accelerations ($\ddot{q}_b$ and $\ddot{q}_d$), a second-order inverse kinematics algorithm should be used [24], although a numerical differentiation of the outputs of algorithm (16) could be adopted in the presence of limited noise and suitably small sampling time. Moreover, if the desired end-effector orientation is assigned in terms of the rotation matrix, $R_{e,d}(t)$, an inverse kinematics algorithm similar to (16) can be adopted, where the kinematic inversion error is computed in terms of the unit quaternion [25].

### B. Motion Control

Once $\zeta_r$ and its derivatives are computed by the inverse kinematics algorithm, they are delivered to the motion control loops in charge of achieving the desired motion. The control strategy proposed here is a generalization of the control laws proposed in [12] for quadrotor vehicles (without the manipulator) to vehicle-manipulator systems. This is, in turn, a hierarchical inner-outer loop control scheme, based...
on two steps. The outer control loop is designed to track the assigned position and yaw angle for the vehicle, by introducing a virtual control vector aimed at linearizing the position dynamics. Then, by using the relation between the force vector \( u_f \) and the quadrotor attitude, a reference value for the roll and pitch angles is devised and used to feed an attitude controller (inner loop). The whole design is possible since the quadrotor position and attitude dynamics can be written as a cascade system in which the attitude subsystem does not depend on the position.

In order to globally linearize the closed-loop dynamics, the following control input \( u \) can be considered

\[
\begin{align*}
\mathbf{u} &= \mathbf{M}(\xi)\alpha + C(\xi, \dot{\xi})\dot{\xi} + g(\xi),
\end{align*}
\]  

where the auxiliary input vector \( \alpha \) can be partitioned, according to (13), i.e., \( \alpha = [\alpha_p^T \alpha_\phi^T \alpha_q^T]^T \), with \( \alpha_\phi = [\alpha_\psi \alpha_\theta \alpha_\varphi]^T \). Hence, the globally linearized closed-loop dynamics is given by \( \dot{\xi} = \alpha \), i.e.,

\[
\begin{align*}
\dot{\mathbf{p}}_b &= \alpha_p, \\
\dot{\mathbf{q}}_b &= \alpha_\phi, \\
\dot{\mathbf{q}} &= \alpha_q.
\end{align*}
\]  

In turn, the auxiliary control inputs for the manipulator, the quadrotor position and the quadrotor yaw angle can be chosen, respectively, as

\[
\alpha_q = \ddot{\mathbf{q}}_r + \mathbf{K}_{q,V}(\mathbf{q}_r - \mathbf{q}) + \mathbf{K}_{q,p}(\mathbf{q}_r - \mathbf{q}),
\]

\[
\begin{align*}
\alpha_p &= \ddot{\mathbf{p}}_r + \mathbf{K}_{p,V}(\mathbf{p}_r - \mathbf{p}) + \mathbf{K}_{p,p}(\mathbf{p}_r - \mathbf{p}),
\end{align*}
\]  

and

\[
\alpha_\psi = \ddot{\mathbf{q}}_r + \mathbf{K}_{\psi,V}(\mathbf{q}_r - \mathbf{q}) + \mathbf{K}_{\psi,p}(\mathbf{q}_r - \mathbf{q}),
\]

where \( \mathbf{K}_{s,V}, \mathbf{K}_{s,p} \) with \( s = \{p, q\} \) are symmetric positive definite gain matrices and \( \mathbf{K}_{q,V}, \mathbf{K}_{q,p}, \mathbf{K}_{p,V}, \mathbf{K}_{p,p} \) are positive scalar gains. If the reference accelerations are not provided by the inverse kinematics algorithms, the acceleration feedforward terms in (20)–(21) could be either omitted or computed numerically (see Remark 1).

1) Quadrotor position controller: On the basis of eq. (17), the following expression of \( u_f \) can be derived

\[
\begin{align*}
\mathbf{u}_f &= \mathbf{M}_{pp}\alpha_p + \mathbf{M}_{pp}\alpha_\phi + \mathbf{M}_{pq}\alpha_q + \mathbf{C}_p\dot{\xi} + g_p,
\end{align*}
\]  

This expression does not allow to compute \( u_f \), since it includes the control \( \alpha_\phi \), whose computation requires references values for roll and pitch angles, not available at this stage. To overcome this problem, \( u_f \) is computed in a slightly different way with respect to (22), i.e.,

\[
\begin{align*}
\mathbf{u}_f &= \mathbf{M}_{pp}\alpha_p + \mathbf{M}_{pq}^{(1)}\alpha_\psi + \mathbf{M}_{pq}\alpha_q + \mathbf{C}_p\dot{\xi} + g_p,
\end{align*}
\]  

where \( \mathbf{M}_{pq}^{(1)} \) denotes the first column of the matrix \( \mathbf{M}_{pq} \). It can be noting that, since the manipulator links are much lighter than the vehicle body, the elements of matrix \( \mathbf{M}_{pq} \) are often negligible with respect to those of \( \mathbf{M}_{pp} \). Therefore, in practice, \( u_f \) in (23) is very close to the ideal control input computed in (22). The validity of this choice will be checked in the simulation case study.

In view of (13), \( u_f \) depends on the attitude of the quadrotor via the relation

\[
\begin{align*}
\mathbf{u}_f = \mathbf{h}(f_z, \mathbf{\sigma}_k) \Rightarrow \begin{bmatrix} u_{f,x} \\ u_{f,y} \\ u_{f,z} \end{bmatrix} &= \begin{bmatrix} (c_\psi s_\theta c_\varphi + s_\psi s_\varphi) f_z \\ (s_\psi s_\theta c_\varphi - c_\psi s_\varphi) f_z \\ c_\theta c_\varphi f_z \end{bmatrix},
\end{align*}
\]  

Therefore, the total thrust, \( f_z \), and reference trajectories for the roll and pitch angles to be fed to the inner loop can be computed as

\[
\begin{align*}
f_z &= \|\mathbf{u}_f\|, \\
\theta_r &= \arctan\left(\frac{u_{f,x} s_\varphi + u_{f,y} c_\varphi}{u_{f,z}}\right), \\
\varphi_r &= \arcsin\left(\frac{u_{f,x} s_\varphi - u_{f,y} c_\varphi}{\|\mathbf{u}_f\|}\right).
\end{align*}
\]  

2) Quadrotor attitude control: Once the reference value for roll and pitch angles have been computed, the control inputs \( \alpha_\theta \) and \( \alpha_\varphi \) can be realized via simple PD control laws

\[
\begin{align*}
\alpha_\theta &= -\dot{\theta} + k_\theta p_r (\theta_r - \theta), \\
\alpha_\varphi &= -\dot{\varphi} + k_\varphi p_r (\varphi_r - \varphi),
\end{align*}
\]  

where \( k_\theta p_r \) and \( k_\varphi p_r \) are positive scalar gains. Finally, \( \mathbf{u}_\mu \) can be computed as

\[
\mathbf{u}_\mu = \mathbf{M}_{p_0}^T \alpha_p + \mathbf{M}_{\phi_0} \alpha_\phi + \mathbf{M}_{\phi_0} \alpha_\psi + \mathbf{C}_\phi \dot{\xi} + \mathbf{g}_\phi.
\]  

and, from (13), the vehicle torques as

\[
\mu_b^h = \mathbf{T}^{-1}(\phi_0)R_0(\phi_0)u_\mu.
\]  

Remark 2: As for \( \alpha_\theta \) and \( \alpha_\varphi \) laws similar to (20) and (19) can be used, requiring a numerical differentiation of \( \theta_r \) and \( \phi_r \), which are likely to be affected by noise.

3) Computation of quadrotor inputs: Once \( f_z \) and \( \mu_b^h \) have been computed, the four actuation forces of the rotors of the vehicle can be easily obtained via (15)

\[
\mathbf{f} = \mathbf{N}^{-1}\begin{bmatrix} f_z \\ \mu_b^h \end{bmatrix}.
\]  

4) Manipulator control: Finally, the torques acting on the manipulator joints can be computed as

\[
\mathbf{u}_q = \mathbf{M}_{q_0}^T \alpha_p + \mathbf{M}_{q_0}^T \alpha_\phi + \mathbf{M}_{q_0} \alpha_\theta + \mathbf{C}_q \dot{\xi} + \mathbf{g}_q.
\]  

IV. SIMULATION RESULTS

The proposed algorithm has been tested in simulation by considering the dynamic model of an ASCTEC PELICAN quadrotor, characterized by mass \( m_0 = 2 \text{kg} \) and inertia matrix \( \mathbf{I}_b = \text{diag}(1.24, 1.24, 2.48) \) [19]. The distance from each motor to the vehicle center of mass, the drag coefficient and the thrust coefficient in (15) have been set to \( l = 0.25 \text{ m} \), \( \gamma_d = 7.5 \cdot 10^{-7} \text{ N s}^2 \), and \( \gamma_t = 3 \cdot 10^{-5} \text{ N s}^2 \), respectively.

The quadrotor is equipped with a 5-DOF robotic manipulator characterized by 5 revolute joints, disposed in such a way that the first two joint axes intersect. The Denavit-Hartenberg parameters [20] of the arm are reported in Table I while Fig. 3 shows the joint axes. The mass and inertia of
the manipulator links have been estimated via a CAD model: the masses are \( m_1 = 80 \text{ g}, m_2 = 10 \text{ g}, m_3 = 6 \text{ g}, m_4 = 2 \text{ g} \) and \( m_5 = 1.5 \text{ g} \), while the moments of inertia of each link through its center of gravity are given by \( I_{l_1} = 1.04 \cdot 10^{-4}, I_{l_2} = 2.27 \cdot 10^{-4}, I_{l_3} = 4.56 \cdot 10^{-5}, I_{l_4} = 7.18 \cdot 10^{-5} \) and \( I_{l_5} = 2.90 \cdot 10^{-6} \).

<table>
<thead>
<tr>
<th>Joint</th>
<th>( d ) [mm]</th>
<th>( \vartheta )</th>
<th>( a ) [mm]</th>
<th>( \alpha ) [rad]</th>
</tr>
</thead>
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<td>( \vartheta_1 )</td>
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<td>0</td>
<td>( \vartheta_2 )</td>
<td>150</td>
<td>( \pi/2)</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( \vartheta_3 )</td>
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</tr>
<tr>
<td>4</td>
<td>0</td>
<td>( \vartheta_4 )</td>
<td>0</td>
<td>(-\pi/2)</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>( \vartheta_5 )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In order to simulate a realistic scenario, only the measurements of position and orientation of the vehicle and the manipulator joint positions have been assumed to be available, while the velocity measurements have been obtained via a first-order filter with a time constant of 0.03 s.

The quadrotor position can be obtained by resorting to GPS (few centimeters accuracy at a rate up to 10 Hz), external localization devices, such as VICON motion capture system (very high accuracy at about 200 Hz) or on-board camera-based systems (few centimeters accuracy at about 20-50 Hz) [26]. As for the attitude measurements, the ASCTEC PELICAN has an inbuilt IMU (Inertial Measurement Unit) equipped with 3 axis gyroscopes, providing the measurements of the attitude rates, 3 axis accelerometers, providing the measurements of the accelerations, and a magnetometer, providing the absolute heading. Then, a filter within the IMU provides an absolute attitude measurement at a rate of 1 KHz [26]. Therefore, in order to simulate the measured data, the following assumptions have been made:

- a normally distributed measurement noise has been added to the position signals; in detail for vehicle position the noise has mean of \( 10^{-3} \) m and standard deviation of \( 5 \cdot 10^{-3} \) m, for vehicle orientation the mean is \( 10^{-3} \) rad and the standard deviation is \( 10^{-3} \) rad, and for the joint position the mean is \( 10^{-4} \) rad and the standard deviation is \( 5 \cdot 10^{-5} \) rad;
- vehicle position and attitude are available at a frequency of 25 Hz;
- manipulator joint position are available at a frequency of 250 Hz.

The controller outputs are computed at a rate of 250 Hz. Finally, in order to test the robustness to the model uncertainties, a nominal estimate of the inertia matrix has been considered, whose elements are assumed to be equal to 0.9 times their true values (i.e., a 10% error).

The simulation model has been built in Matlab/Simulink\textsuperscript{©} environment. The controller parameters are summarized in Table II.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{p,p} )</td>
<td>1.5 ( \cdot ) ( I_g )</td>
<td>( K_{p,V} )</td>
<td>2.5 ( \cdot ) ( I_g )</td>
</tr>
<tr>
<td>( K_{q,q} )</td>
<td>20 ( \cdot ) ( I_g )</td>
<td>( K_{q,V} )</td>
<td>5 ( \cdot ) ( I_g )</td>
</tr>
<tr>
<td>( k_{\vartheta,p} )</td>
<td>10</td>
<td>( k_{\vartheta,V} )</td>
<td>5</td>
</tr>
<tr>
<td>( k_{\varphi,p} )</td>
<td>12.5</td>
<td>( k_{\varphi,V} )</td>
<td>5</td>
</tr>
<tr>
<td>( K )</td>
<td>50 ( \cdot ) ( I_g )</td>
<td></td>
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</tbody>
</table>

The end-effector is tasked to follow a circular helix with radius of 1.5 m and pitch of \( \pi/2 \) (Fig. 4) and, then, an hovering phase of 5 s is commanded. Moreover, rotations of the end-effector of \( \pi/4 \) rad along roll, pitch and yaw axis is required.

Figs. 5–9 report the simulation results. In detail, Fig. 5 shows the norm of end-effector position and orientation errors between the desired and the planned values. The noise affecting the data are due to the term \( \sigma_\theta \) in (16). As for controller performance, Fig. 6 reports both the desired path of the manipulator end-effector and the actual one, expressed in the fixed frame. The position and orientation errors are shown in Fig. 7, it can be seen that a good trajectory tracking is achieved; during the transient the maximum error is about \( 4 \cdot 10^{-2} \) m and \( 4 \cdot 10^{-2} \) rad, while at steady state the residual errors are of the same order of magnitude of the measurement noise mean. Finally, Fig. 8 and Fig. 9 show the control efforts. In detail, Fig. 8 shows the thrust of the single rotors of the vehicle, while Fig. 9 shows the joint torques of the robotic arm.

Fig. 3. Robot manipulator.

Fig. 4. 3D trajectory of the end-effector.
Finally, the validity of the assumption done in Section III-B.1 is checked. To this aim, the norm of the inertia matrix $M$ and the norm of the matrix $M_0$, obtained by setting to zero the second and third column of the sub-matrix $M_{p0}$ in $M$, have been compared during the simulation. The results, both in terms of absolute values and differences, have been reported in Figs. 10(a) and 10(b), respectively. It can be seen that the norms are very close, therefore it can be argued that the use of $M_0$ for control design in lieu of $M$ is an acceptable assumption.

Remark 3: The proposed controller, has been tested also in ideal conditions, i.e., in the absence of uncertainties and assuming a frequency of 1 KHz for all the measured data. In this case the error peaks are one order of magnitude lower and of the same order of magnitude of the noise. The figures referred to this simulation are not reported for the sake of brevity.

V. CONCLUSIONS AND FUTURE WORK

In this paper, a novel hierarchical motion control scheme has been proposed for a quadrotor aerial vehicle equipped with a robotic arm. First, on the basis of the desired pose of the arm end-effector, an inverse kinematics algorithm computes the references for the vehicle position, the vehicle yaw angle and for the manipulator joints. Then, a cascade controller ensures the tracking of the desired trajectories. A simulation case study proves the effectiveness of the approach also in the presence of nonidealities and model uncertainties. Future work will be focused on a rigorous stability analysis and on the experimental validation of the presented results.

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Fig. 9. Joint torques for the robotic arm.

(a) torques of joints 1–3
(b) torques of joints 4–5

Fig. 10. Comparison between the norm of the inertia matrix and the norm of the matrix $M_0$ used for control design.

(a) absolute values
(b) $\|M\| - \|M_0\|$


